Droplet Formation from a Liquid Film over a Rotating Cylinder

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The interfacial instability in the molten layer on a vehicle entering the atmosphere at great speeds is discussed. This instability, of the Taylor type, leads to the formation of spikes from which droplets break away. Experimental results on a rotating cylinder simulating re-entry deceleration reveal two distinct phases of droplet development. First, a threshold condition for the formation of spikes is reached. This condition represents a minimum logarithmic increment for the prevailing unstable disturbances and is found to be between 7 and 10 for the present experiments. The magnitude has been approximated from the relation $Gh/10\nu$, where G is the relative gravitational field, h the film thickness, and ν the kinematic viscosity. The second phase consists of the droplet formation. Droplet sizes have been correlated by means of the relation $d = 2(3)^{1/2}\ (T/\rho G)^{1/2}$ obtained by drop-tip analogy, where d is the diameter, and T, ρ, and G the surface tension, density, and relative acceleration, respectively. The tip mass and the droplet appear to fall from the surface with a constant acceleration of about 0.8 g. According to the present results, a re-entering satellite reactor residue may give rise to droplets of slightly over 1-mm diam under typical conditons.

Introduction

When a body enters the dense strata in the atmosphere from space at a very high velocity, the body surface melts, and the material is dispersed along its trajectory. The fineness of the dispersed particles is of practical concern when the constituent of the body is a radioactive fuel or residue. This concern has added importance to the earlier academic interest on the phenomenon of droplet formation.

The severe aerodynamic heating on a re-entry body produces a molten layer over the surface of the body, and the aerodynamic drag causes the body to decelerate. The region of air-liquid interface with effective gravitational force field directed windwardly is unstable. It is conjectured in Ref. 1 that this instability, of the Taylor type, leads to the formation of droplets breaking away from the molten layer. The droplets may or may not disintegrate further, depending on the prevailing aerodynamic circumstances and the size of the initial droplets. The main concern here is the presence of a threshold condition for the shedding of droplets and of a correlation of the size of the droplets breaking away from the liquid layer over the extended surface of a rotating cylinder.

Earlier Experiments

Different aspects of the mechanics of drops are reviewed in Ref. 2, and the atomization of liquid fuels by different methods is reviewed in Ref. 3. Of the experiments reviewed therein, the one concerning the production of drops from the rim of a rotating disk is of interest in the present discussion. In such tests, when the liquid is fed over both faces of the disk from the center at an appropriate rate with both edges of the disk covered with liquid, pendent drops form, attached to the rim by individual stems. The pendent drops eventually break away from their stems forming primary drops with diameters nearly the same as the thickness of the liquid rim (not the disk thickness). The long stems of liquid subsequently break up into satellite droplets of smaller sizes. Experimental data on the diameter of the primary drops are correlated fairly well by

$$d = 2.3\ (T/\rho G)^{1/2} \tag{1}$$

where d is the diameter of the drop, and T and ρ are the surface tension and density, respectively, all in cgs units; G is the effective gravitational acceleration ω^2R at the rim of the disk of radius R(cm), rotating at an angular velocity of ω(rad/sec).

The dimensional form may be anticipated from either the static consideration of the drop-tip analogy or Rayleigh's instability theory of a liquid column.[2] Correlation (1) for the rotating disk is relatively insensitive to the liquid viscosity over the range of 1 to 1500 cp. The edge profile of the disk influences significantly the droplet size under otherwise similar conditions (see Ref. 3); in a thick disk, the liquid may not form a rim but may instead break away from the two edges; thus, correlation (1) will no longer apply.

Present Experiments

The instability theory of a liquid film (see Ref. 1) can, at best, suggest the dimensional dependence of the droplet size just as the Rayleigh's instability theory does for the case of the breakup of a liquid thread. Consequently, a rotating-cylinder experiment has been carried out to find an empirical quantitative correlation of the droplet size and any other aspects of interest.

In these tests, a thin liquid film was maintained on the porous surface element of a rotating cylinder with the liquid supplied from a hollow annulus. In this manner, the liquid film is thin and controllable by the liquid supply. The lateral extent of the interface was very large, so that little edge-profile effect, if any, was expected.

The axis of the rotating cylinder was carefully aligned with the terrestrial gravity and the whole apparatus rigidly constructed and anchored to minimize vibration. The

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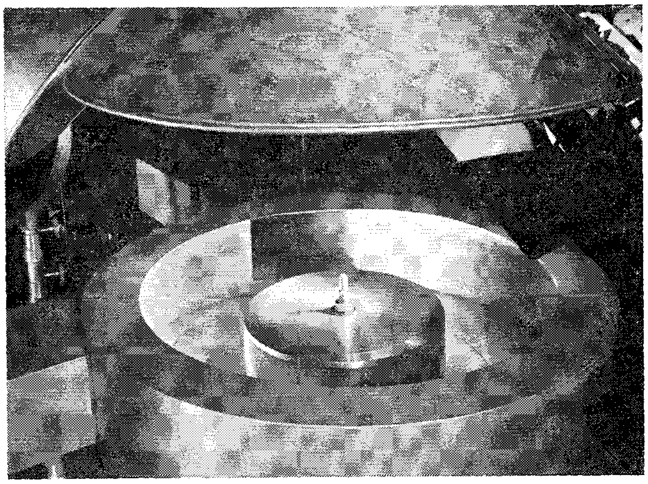

Fig. 1 View of droplet test apparatus.

porous element was fabricated of sintered bronze with the characteristic porosity small enough relative to the film thickness to produce uniform transpiration.

Figure 1 shows the test apparatus. Some typical results, recorded by single flash photography, are shown in Figs. 2–6. The physical properties of the test liquids are listed in Table 1.

The general features of droplet formation from a rotating cylinder are similar to those from a rotating disk. Primary drops are formed of more or less uniform size, attached to the liquid film with individual threads. The primary drops eventually break away, and the threads disintegrate into smaller satellite droplets.

Figure 7 presents a correlation of the diameters of the primary droplets from the present tests with the capillary parameter $(T/\rho G)^{1/2}$. All physical quantities are in cgs units. The data points have been obtained from tests conducted under a variety of conditions, including variations in kinematic viscosity and the flow rate of liquid to the film. Increased flow rate resulted in more copious drops but did not alter significantly the physical dimensions of the phenomena. Within the experimental scatter, the data are very well correlated by

$$d = 2 \times (3)^{1/2} (T/\rho G)^{1/2} \tag{2}$$

independent of liquid viscosity. The numerical coefficient $2 \times (3)^{1/2}$ is obtained from the following simple static consideration in analogy to the drop hanging from a tip.

On the verge of forming a cylindrical stem, the surface tension is just enough to balance the gravitational pull on a semispherical tip. At any station on the cylindrical stem, the gravitational pull on the additional mass cannot be balanced; so, the pendent drop accelerates in the direction of the effective gravitational pull. The requirement of mass continuity of the liquid will bring about the accelerated narrowing down of the cylindrical stem and the rapid formation of the waist, the neck, and the thread in succession.

Table 1 Properties of test liquids

Liquid	ν, centistokes	T/ρ, cgs	Temperature, °F
Water	1	73.16	70
Ethyl alcohol	1.5	28.66	70
Way-D oil[a]	100	32.08	87
URSA oil[a]	220	30.92	113
URSA oil	250	31.85	100
URSA oil	380	32.20	93
URSA oil	470	32.60	86

[a] Way-D and URSA are Texaco trade names.

The different coefficients in correlations (1) and (2) may be attributed to the edge restraint in the rotating disk. The apparent absence of any dependence on liquid viscosity is in favor of the static consideration of the drop-tip analogy.

As the effective gravitational field increases in the present series of experiments, the droplet size decreases linearly but appears to reach a minimum of about 0.12 cm when the capillary parameter $(T/\rho G)^{1/2}$ is reduced to about 0.035. Two tests performed at a smaller capillary parameter gave off drops of sizes much larger than the minimum size. The deviation from the linear extrapolation of the correlation appears too large to be accounted for by the uncertainties in the measurements; yet it is premature to conclude from this bit of evidence that some dynamic aspect becomes important when the capillary parameter is less than 0.035.

Near the other extreme of the tested range of the capillary parameter, the failure of correlation (2) assumes a different

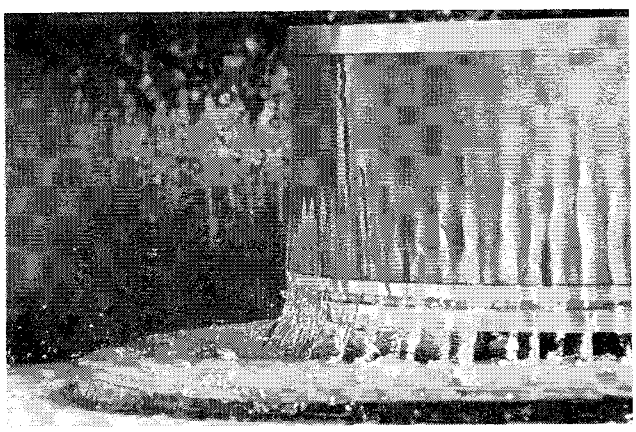

Fig. 2 Ethyl alcohol on cylindrical element (120 rpm, radial acceleration 2736 cm/sec²).

Fig. 3 Ethyl alcohol on cylindrical element (216 rpm, radial acceleration 7797 cm/sec²).

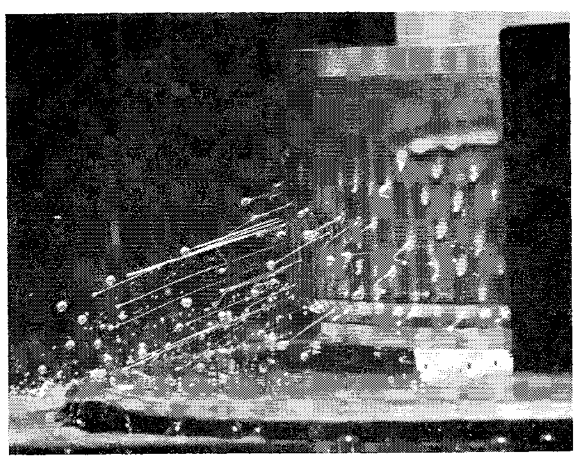

Fig. 4 URSA oil on cylindrical element (150 rpm, radial acceleration 3756 cm/sec²).

form. Figure 2 shows the experiment with ethyl alcohol film on a rotating cylinder with a value of the capillary parameter of 0.0995. Correlation (2) would predict, under these conditions, the shedding of droplets of about 0.35 cm in diameter. However, none are observed. Likewise, no drops were formed for ethyl alcohol at 0.0788 of the reference parameter, or for URSA oil at 0.122. These points are indicated in Fig. 7. Closer observation of the test results reveals that the failure of the shedding of droplets is not limited to the larger values of the capillary parameters in the tested range.

From this observation, it is apparent that the shedding of any droplets from the liquid film must be preceded by the fulfillment of some threshold condition. The formation of droplets from a liquid film may be considered to develop through two stages.

The first is the dynamic amplification of the unstable interfacial waves (the Taylor-type instability), to provide partially developed spikes as bases for the forming drops. The second is the parting of the spike heads from the bases to form pendent drops as is in the drop-tip analogy.

The transition from the first to the second stage is marked by the fact that the surface tension at some section across the partially developed spike becomes insufficient to balance the effective gravitational pull on the tip mass. The threshold condition for the formation of droplets in tests at a given noise level must then be some minimum logarithmic increment of the interfacial disturbances.

For any interface of a limited extent, the time interval available for any unstable wave component to amplify is correspondingly limited. If the most unstable component of the disturbance cannot reach the minimum logarithmic increment during the limited time interval, no droplets will form. Thus, such physical quantities as film thickness, liquid viscosity, and liquid supply rate, among others, including the capillary parameter itself, which influence the amplification rate of the disturbances either directly or indirectly, will bear on the question of whether or not any droplets will form and shed.

When the physical conditions in a series of tests are varied in such a way as to increase the amplication rates of the disturbances, the threshold condition is eventually reached by some disturbance component, and droplets begin to form. Further increase of the amplification rates will bring wider ranges of the unstable components of the disturbances to fulfill the threshold condition and provide more numerous bases for the pendent droplets.

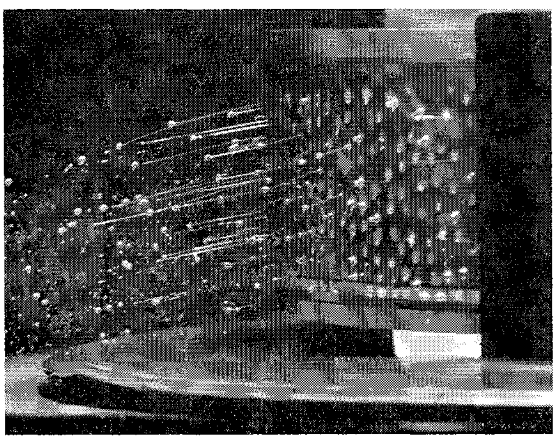

Fig. 5 URSA oil on cylindrical element (190 rpm, radial acceleration 6035 cm/sec²).

Fig. 6 Way-D oil on cylindrical element (140 rpm, radial acceleration 3275 cm/sec²).

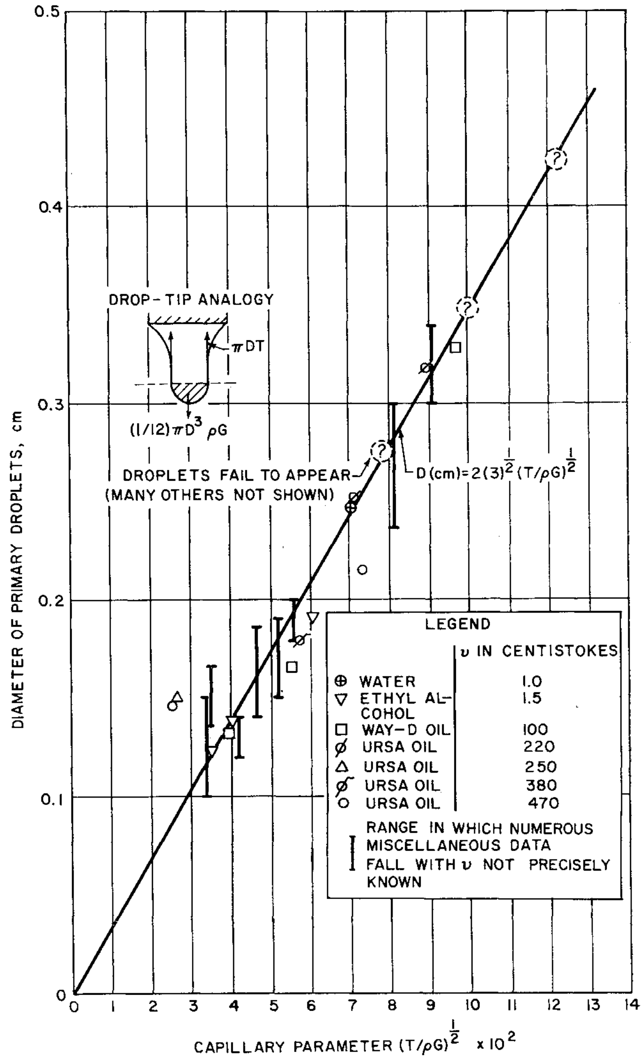

Fig. 7 Diameter of primary droplets vs capillary parameter.

It is clear in Figs. 2–6 that the number of droplets shed increases toward the lower edge of the cylinder where more time is available for amplification. As an alternate verification, a set of experiments were conducted with URSA oil in which the supply line pressure was varied from 8 to 36 oz/in.². The higher rate of liquid supply resulted in more copious shedding rates but without noticeable changes in drop size and other test conditions.

With information obtained from the photographs of the tests reported here, it is possible to estimate the minimum logarithmic increment for droplet formation. The film thickness h at the location where the pendent drops first appear is measured on the photographs. The local amplification rate is approximated by $Gh/10\nu$, as suggested in Ref. 1.

The critical minimum logarithmic increment for the reported test conditions is found to be between 7 and 10.

Such a large logarithmic increment indicates that, when the base of the pendent drop begins to form, the Taylor instability has developed far into the nonlinear stage of spike formation. The same is suggested by the following approximate analysis of the motion of the droplets.

In Figs. 2–6, the projections of the stems in the x-z plane (the plane of the paper) are essentially parallel straight lines with slopes greater than $g/\omega^2 R$, where g is the earth's gravitational acceleration. Neglecting aerodynamic drag, an analysis of the motion of the droplet relative to the root of the spike has been made (see the Appendix). From this analysis, it is concluded that 1) the drop must leave the mean interface of the liquid film with an initial radial velocity component $U_r = (\Delta x/\Delta z) U_z$, where $\Delta z/\Delta x$ is the slope of the stem, and U_z is the z-velocity component of the liquid mass on the interface due to the earth's gravity; and 2) the drop must leave the mean interface with a radial acceleration (in the rotating framework) of approximately $\frac{4}{5}$ of the centrifugal acceleration $\omega^2 R$.

The fact that correlation (2), deduced from a purely static consideration, agrees quantitatively with the data, is, hence, a coincidence. From the analysis of the droplet motion described in the Appendix, the effective acceleration to be used while applying the drop-tip analogy should be about 0.8

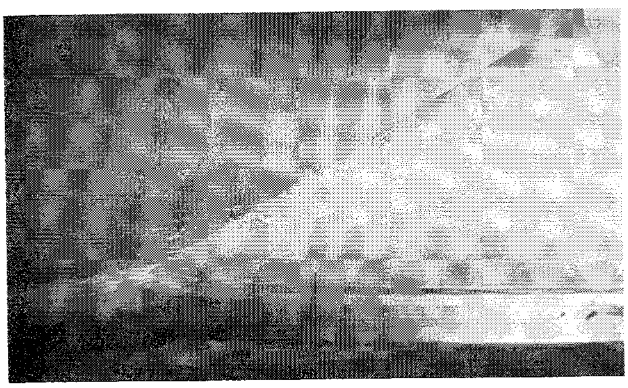

Fig. 8 Way-D oil on 60° element (680 rpm, radial acceleration at critical point of droplet separation 76,000 cm/sec^2).

of the centrifugal acceleration $\omega^2 R$, not $\omega^2 R$ itself. Nevertheless, the tip of the spike that separates off is clearly not spherical. The mass of the pendent drop is larger than that in the spherical cap. In the balance of forces, this additional mass compensates for the reduced effective acceleration and any excess or defect of the static pressure in the liquid behind the deforming-curved free surface from the surrounding atmospheric pressure.

The presence of this initial radial velocity and radial acceleration of the pendent drop also indicates the advanced state of development of the interfacial waves under Taylor instability when the drop-tip analogy takes over.

It seems fair to conclude that the size of the droplets estimated from correlation (2) is reasonably accurate when droplets are shed. A threshold condition determines whether or not droplets are shed. The threshold condition appears to be a minimum logarithmic increment for the prevailing noise level. Whether or not any droplets are shed will, hence, depend on the prevailing amplication rate of the interfacial disturbances of the liquid film and the time interval available for the unstable component to grow.

The experiments reported herein are of exploratory nature. More refined measurements and finer control of the experimental parameters would permit a closer examination of the threshold condition; also, they would allow a better evaluation of the effects of film thickness, the rate of liquid supply (the latter corresponding to the rate of melting of the solid core of the re-entering body), and other variables on the threshold condition.

Application

The deceleration that will be encountered by a reactor fuel element after exposure from the cannister of a satellite on a typical decay trajectory is about 7 g near peak heating. If the re-entering body is zirconium with 10% uranium alloy, for which the surface tension has been measured as 1300 to 1370 dynes/cm, the value of the parameter (T/ρ) is approximately 220 cm^3/sec^2. Hence, the magnitude of the capillary parameter $(T/\rho G)^{1/2}$ is 0.032 or larger. According to Fig. 7, droplets of this zirconium-uranium alloy of about 1 to 2 mm in diameter may be expected. The variation of the droplet size depends largely on the prevailing deceleration when the molten film forms on the surface of the body.

Effects from Other Than Normal Gravitational Field

Since the interfacial waves ride over the interfacial mass motion, an increase of the velocity of the mean mass motion along the surface will decrease the time available for a given unstable disturbance to amplify while traveling a given distance along the surface. In order to reach a given minimum logarithmic increment to provide the bases for shedding droplets, the amplification rate of the interfacial disturbances (proportional to the effective gravitational field) must be correspondingly increased.

Over the surface of a rotating cone, the interfacial mass is accelerated by a component of the centrifugal acceleration which is several times larger than the terrestrial g acting along the cylindrical surface in the tests just reported. Hence, other conditions being equal, the shedding of droplets from a wetted rotating conical surface will begin at a larger effective centrifugal force field than that from a rotating cylinder. This qualitative expectation and others are in agreement with limited experimental data from rotating cones of 45° and 60° semivertex angle. Figure 8 shows shedding of droplets from a 60° element on which droplets begin to form at a location along the conical surface, where the radial acceleration is 76,000 cm/sec^2. An analysis of the results from this experimental situation, which corresponds to the conditions away from the stagnation region of a re-entering body, will have to be reported separately.

Appendix

A general feature in Figs. 2–6 is that the projection of the stems attached to the liquid film within, say, 20° from the plane of the paper appears to be a system of parallel straight lines. This kinematic property offers an opportunity to infer some dynamic aspects of the system.

Let z be the axis of rotation, x-z be the plane of the paper, and $r^2 = x^2 + y^2$. Consider a drop that began to detach from the liquid film at the angular position ϕ from the x-z plane with a tangential velocity ωR, radial velocity U_r, axial velocity U_z, and a constant radial acceleration $A_r = a_r + \omega^2 R$ with respect to the rotating cylinder. The projection of its stem in the x-z plane at instant t later is given by

$$\Delta x = -R\omega t \sin\phi + [U_r t + (a_r/2) t^2] \cos\phi - R [\cos(\phi + \omega t) - \cos\phi]$$

$$\Delta z = U_z t + \tfrac{1}{2} g t^2$$

In the limit of $t \to 0$ at any ϕ, $(\Delta x/\Delta z)_{t=0} = (U_r/U_z) \cos\phi$. In the limit of $\phi \to 0$ at any t,

$$\left(\frac{\Delta x}{\Delta z}\right)_{\phi=0} \cong \left(\frac{\Delta x}{\Delta z}\right)_{t=\phi=0} = \frac{a_r + \omega^2 R}{g} = \frac{A_r}{g}$$

For arbitrary positive values of ωt and ϕ,

$$\frac{\Delta x}{\Delta z} = \left(\frac{\Delta x}{\Delta z}\right)_{\phi=0}\cos\phi + \left[\frac{\omega^2 R/g}{(\omega t/2) + (\omega U_z/g)}\right] \times \left\{\left(\frac{\sin\omega t}{\omega t} - 1\right)\sin\phi + \left(\frac{4\sin^2(\omega t/2)}{\omega^2 t^2} - 1\right)\frac{\omega t}{2}\cos\phi\right\}$$

is obtained. If $|\phi| \sim |\omega t + \phi| < \frac{1}{3}$ rad, the term in the braces is less than 10^{-2}, and the correction term to $(\Delta x/\Delta z)_0$ is less than 5%, a relatively insignificant correction.

Hence, the kinematic description of the motion is consistent with observations.

The radial acceleration A_r with which a drop falls away from the rotating surface is given as $A_r = (\Delta x/\Delta z)(g/\omega^2 R) \times \omega^2 R$. When $\Delta x/\Delta z$ and $g/\omega^2 R$ are taken from the test values, $A_r \cong 0.8\,\omega^2 R$ is found.

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Liquid Sloshing in a Cylindrical Quarter Tank

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With the increasing size of space vehicles and their larger diameter, both of which lower the natural frequencies of the propellants, the effects of propellant sloshing upon the stability of the vehicle are becoming more critical, especially since at launch usually a very large amount of the total mass is in the form of liquid propellant. With increasing diameter, the oscillating propellant masses and the corresponding forces increase. Furthermore, the natural frequencies of the propellant become smaller and shift closer to the control frequency of the space vehicle. A relatively simple means of avoiding strong dynamic coupling of the propellant motion and the control system is represented by compartmentation of propellant containers with longitudinal walls. This results in smaller sloshing masses and larger natural frequencies. Free and forced liquid oscillations in form of translatory, pitching, and roll excitation have been determined for a cylindrical container of circular quarter cross section. The fluid was assumed to be irrotational, inviscid, and incompressible. The velocity potential of the liquid is obtained from the solution of Laplace's equation with linearized boundary conditions. Forces and moments of the liquid are obtained by integration of the pressure distribution along the container walls. The results of the theoretical studies compared with available experimental values are in good agreement.

I. Introduction

IN space boosters, the diameter of the propellant tanks becomes rather large, and the response of the vehicle to the motions of the container liquid will greatly affect the stability and control of the space vehicle. Propellant oscillations are important because there is a possibility of extreme amplitudes if the excitation frequency is in the neighborhood of one of the natural modes of the propellant. Since a very large amount of the total weight of the vehicle is in the form of liquid propellant, the influence of propellant sloshing upon the stability of the vehicle becomes more critical with increasing tank diameter. The close grouping of control frequency and natural frequencies of the propellant, the relatively low structural frequencies, the very rapid increase of the oscillating propellant masses, and propellant forces with increasing diameter demand thorough investigation of this phenomenon.

The problem of free fluid oscillations in a circular cylindrical container was treated in 1829 by Poisson. Because the theory of Bessel functions was unavailable at the time, the result was not completely interpreted.[1] In 1876, Rayleigh[2] gave the solution for free oscillations in rectangular and cylindrical tanks of circular cross section. In recent years, the problem of forced fluid oscillations has grown in importance.[3] Graham and Rodrigues[4] determined the forced vibration of liquid in rectangular containers, whereas[5] Lorell gave the flow of a fluid in a two-dimensional rectangular container and cylindrical tank of circular cross section for translational excitation. Almost at the same time, many reports appeared about forced fluid oscillations in cylindrical tanks.[6–9] Fluid oscillations in cylindrical tanks with annular[10] and elliptic[11] cross section as well as those in horizontal circular cylindrical tanks and spherical containers[12] have also been treated.

The natural frequencies of the propellant in cylindrical tanks with circular cross section is proportional to the inverse of the root of the tank diameter, whereas the sloshing masses exhibit rather large magnitudes.

To eliminate the unfavorable effect of the propellant motion upon the stability of a space vehicle, various measures can be explored. Internal damping in the liquid can be introduced by fixed baffles[13] or movable slosh suppression devices such as rigid lids following the free propellant surface or floating bodies partially submerged below the free fluid surface. However, use of moving parts is usually avoided because of structural and weight reasons.

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